

Monotonic Target Assignment For Robotic Networks

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The Target Assignment Problem

- n mobile robots, m target locations, in \mathbb{R}^2
- Each robot with unique identifier
 - moves $\dot{p}^{[i]} = u^{[i]}$, $|u^{[i]}| \leq v_{\max}$
 - knows, or can sense, the target locations
- Discrete-time communication model
 - communication range r_{comm}
 - max message length $O(\log n)$

Problem: distributed algorithm to

- allow group of agents to divide m targets among themselves;
- lead each agent to its unique target in minimum time.

Centralized assignment problems:

- Max. matching in bipartite graphs (Hopcroft and Karp, '73)
- Sum assignment problem (Kuhn, '55)
- Bottleneck assignment problem (Derigs and Zimmermann '78)

Parallel/Decentralized assignment problems

- The auction algorithm (Bertsekas, '88)
- Others include Zavlanos and Pappas, Castañón and Wu, Moore and Passino, Arslan, Marden and Shamma.

Distributed Target Assignment

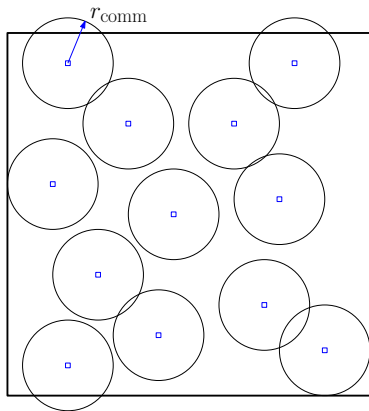
Our Goals:

- Develop efficient algorithms for target assignment problem.
- Evaluate scalability/asymptotic performance.

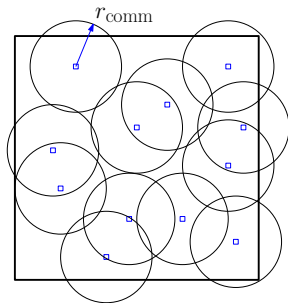
Key Challenge: Optimize **completion time** while satisfying

- ① **range constraint**: compute distributed assignment, possibly without connectivity.
- ② **bandwidth constraint**: share assignment data sparingly.

Size of Environment $\mathcal{E}(n)$ as $n \rightarrow +\infty$



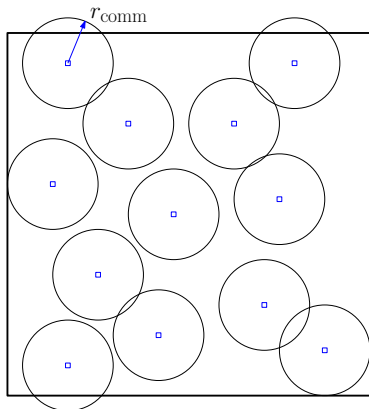
Sparse: $|\mathcal{E}(n)|/n \rightarrow +\infty$



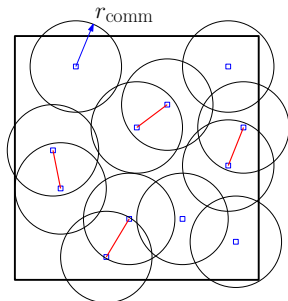
Dense: $|\mathcal{E}(n)|/n \rightarrow 0$

Critical: $|\mathcal{E}(n)|/n \rightarrow C \in \mathbb{R}_{>0}$

Size of Environment $\mathcal{E}(n)$ as $n \rightarrow +\infty$



Sparse: $|\mathcal{E}(n)|/n \rightarrow +\infty$



Dense: $|\mathcal{E}(n)|/n \rightarrow 0$

Critical: $|\mathcal{E}(n)|/n \rightarrow C \in \mathbb{R}_{>0}$

Monotonic Algorithms for Target Assignment

Definition (Monotonic algorithms)

- deterministic algorithm
- target j occupied at time $t_1 \Rightarrow$ target j occupied for all $t > t_1$.

Theorem (Worst-case lower bound on Monotonic Algs)

n agents and n targets in square $\mathcal{E}(n)$:

$\mathcal{E}(n)$ size	Worst-case completion time
Sparse $\left(\frac{ \mathcal{E}(n) }{n} \rightarrow +\infty\right)$	$\Omega(\sqrt{n \mathcal{E}(n) })$
Critical $\left(\frac{ \mathcal{E}(n) }{n} \rightarrow C\right)$	$\Omega(n)$
Dense $\left(\frac{ \mathcal{E}(n) }{n} \rightarrow 0\right)$	$\Omega(\mathcal{E}(n))$

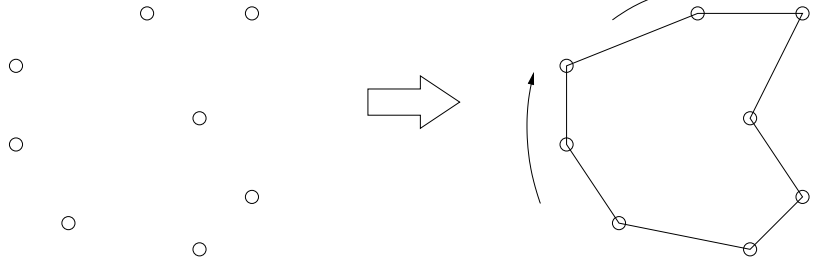
ETSP ASSGMT for Sparse Environments

- Target locations known **a priori**
- Maintain “available/taken” bit for each target.

The Basic Ideas:

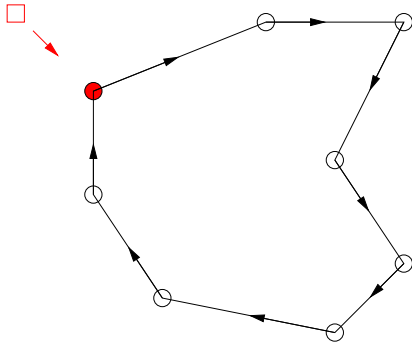
- ① all agents turn the cloud of targets into **ordered ring**
- ② move toward the **closest** target on the ring
- ③ if agent loses conflict, move to **next available** target on ring
- ④ agents exchange **segments of tour** that are “taken.”

Idea 1: create ordered ring



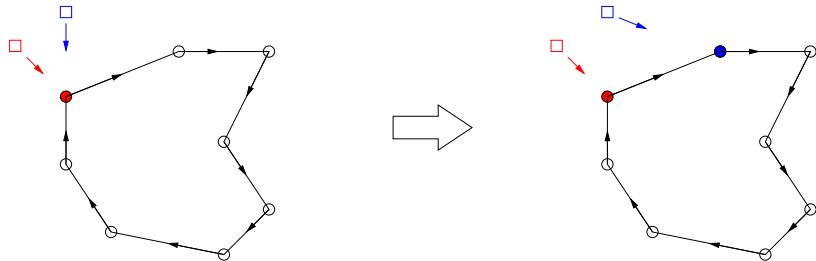
- constant factor ETSP
- same tour for all agents, same order

Idea 2: move toward the closest target on ring



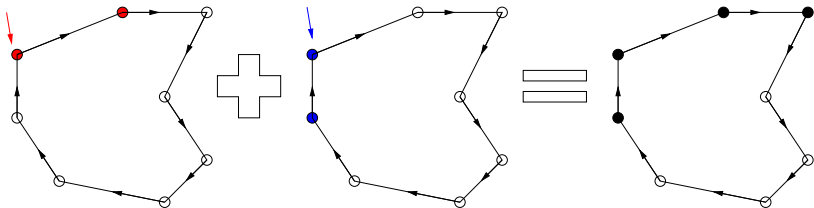
Agent keeps “current” pointer and moves accordingly

Idea 3: lose conflict, move to next available target on ring



- closest agent wins conflict,
- loser selects next target on ring which may be available.

Idea 4: transmit a segment of the tour



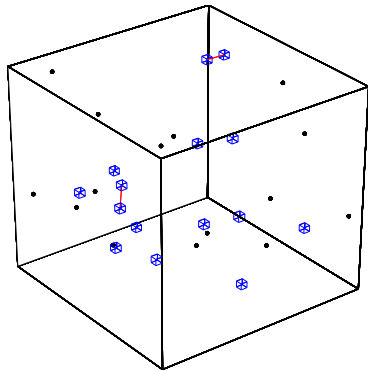
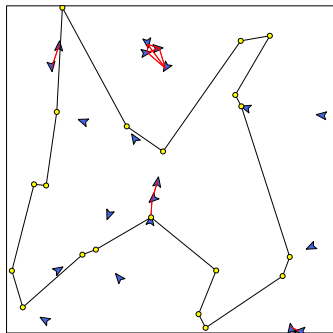
- message transmission $O(\log n)$ bits.
- merge “taken” segments.

Theorem (Worst-case upper bound)

- Assume n agents, n targets in $\mathcal{E}(n)$,
- then worst-case completion time in $O(\sqrt{|\mathcal{E}(n)|n})$.

Sparse/critical $\mathcal{E}(n) \Rightarrow$ ETSP ASSGMT is an **asymptotically optimal** monotonic algorithm

Simulations for ETSP ASSGMT



GRID ASSGMT Algorithm for Dense Environments

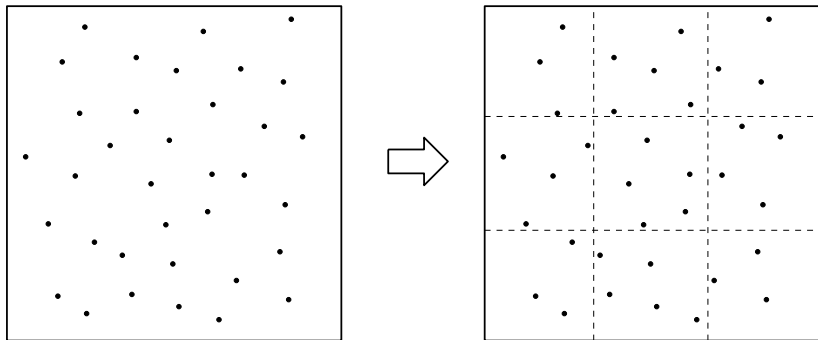
The Basic Ideas:

- 1 All agents partition environment into **small cells**.
- 2 In each cell, agents find **maximum matching** and elect **leader**.
- 3 **Leaders communicate** to determine location of free targets.
- 4 Unassigned agents are **directed to free targets** by leaders.

Assumes either

- Each agent knows target locations *a priori*, or
- no *a priori* knowledge but $r_{\text{sense}} \geq \sqrt{2/5}r_{\text{comm}}$ to **sense**.

Idea 1: partition the environment

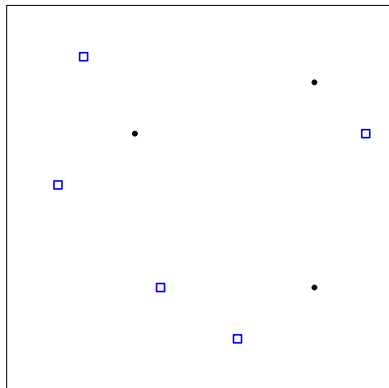


Choose grid size, based on $\mathcal{E}(n)$ and r_{comm} so that:

- Communication graph in a cell is complete.
- Communication between adjacent cells is possible.

Idea 2: leader election and maximum matching in each cell

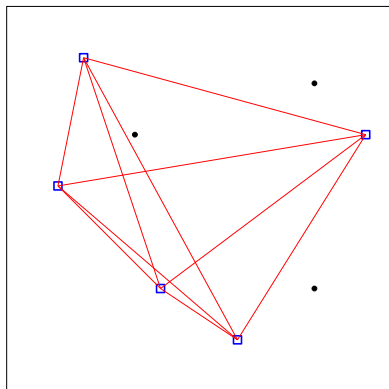
- Match agents to targets
- Elect leader



Example cell

Idea 2: leader election and maximum matching in each cell

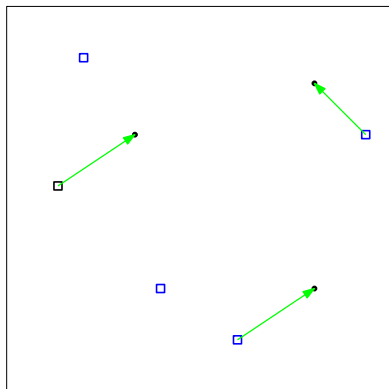
- Match agents to targets
- Elect leader



Example cell

Idea 2: leader election and maximum matching in each cell

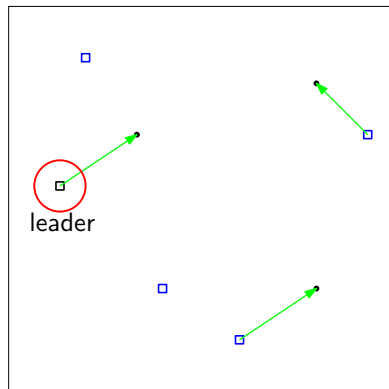
- Match agents to targets
- Elect leader



Example cell

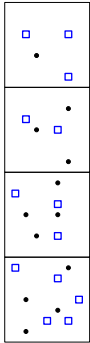
Idea 2: leader election and maximum matching in each cell

- Match agents to targets
- Elect leader



Example cell

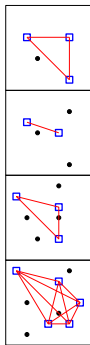
Idea 3: leaders estimate free target locations



Example column

Leaders estimate number of available targets.

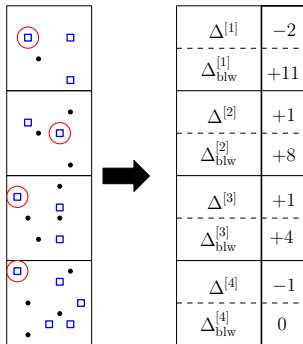
Idea 3: leaders estimate free target locations



Example column

Electing leader in each cell

Idea 3: leaders estimate free target locations

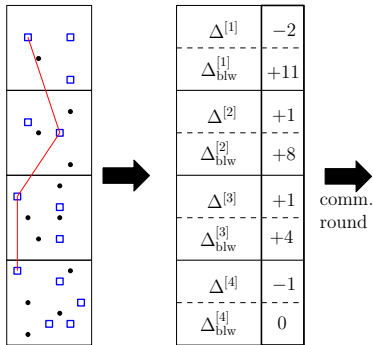


Example column

Initialization

$$\Delta^{[i]} = (\bullet - \square) \text{ in cell } i \quad \Delta_{\text{blw}}^{[i]} = \text{est. of } (\bullet - \square) \text{ in cells below } i$$

Idea 3: leaders estimate free target locations

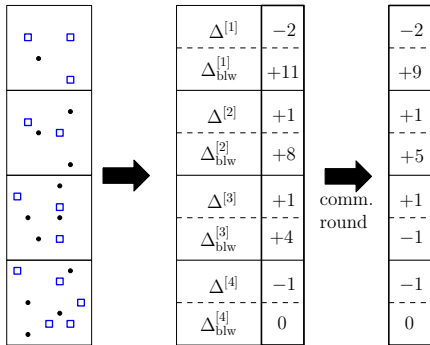


Example column

Initialization

Update estimates through communication

Idea 3: leaders estimate free target locations

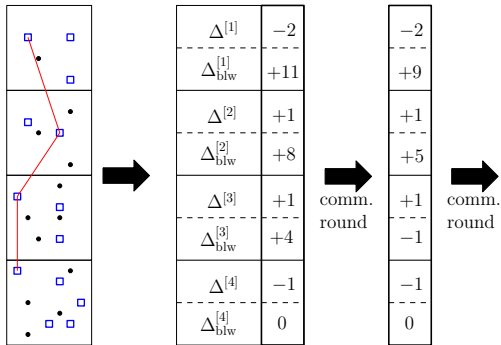


Example column

Initialization

Update estimates through communication

Idea 3: leaders estimate free target locations

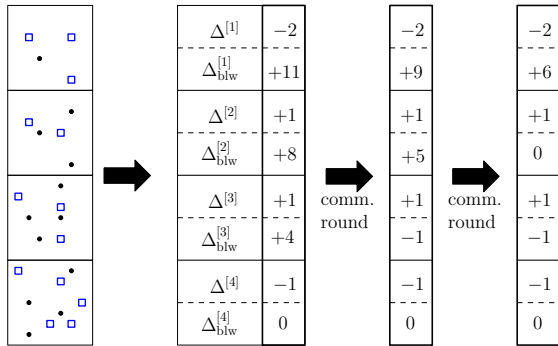


Example column

Initialization

Update estimates through communication

Idea 3: leaders estimate free target locations

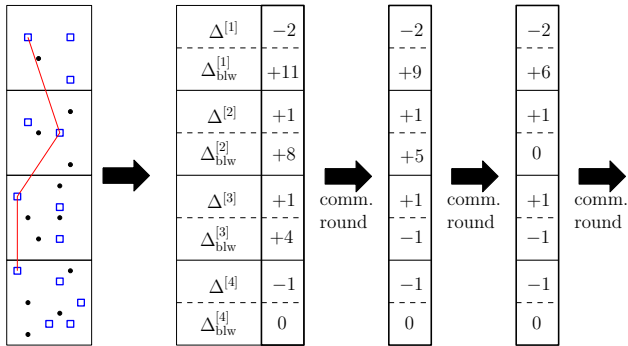


Example column

Initialization

Update estimates through communication

Idea 3: leaders estimate free target locations

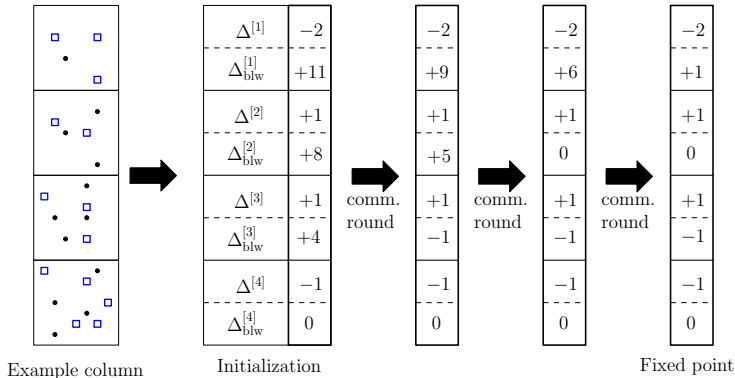


Example column

Initialization

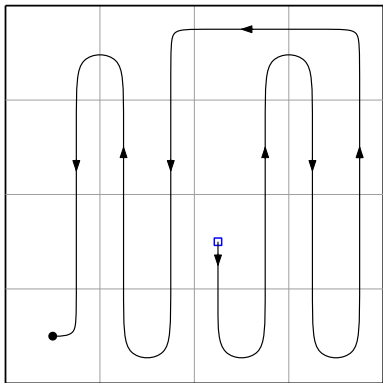
Update estimates through communication

Idea 3: leaders estimate free target locations

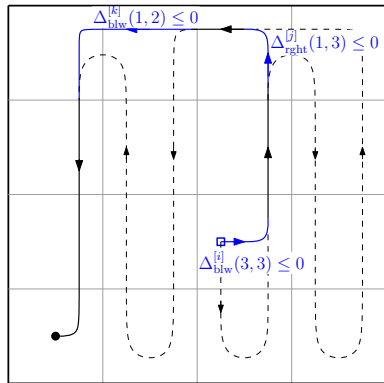


Only let unassigned agents “down” if estimates are positive

Idea 4: unassigned agent motion

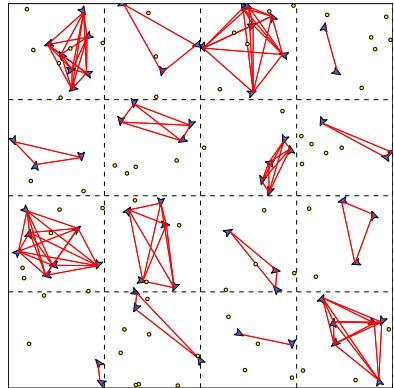
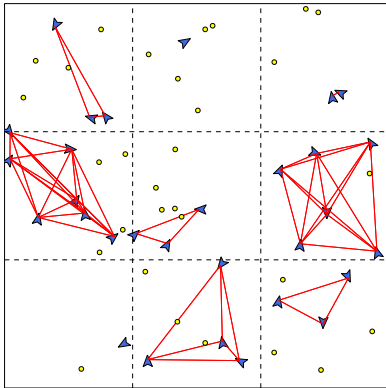


Nominal search order



Search order with leader comm.

GRID ASSGMT Simulations



Worst-case upper bound for GRID ASSGMT

Theorem (Worst-case upper bound)

- Assume n agents, n targets in $\mathcal{E}(n)$.
- then worst-case completion time in $O(|\mathcal{E}(n)|)$.

Worst-case performance comparison:

	Sparse	Critical	Dense
Monotonic	$\Omega(\sqrt{ \mathcal{E}(n) n})$	$\Omega(n)$	$\Omega(\mathcal{E}(n))$
ETSP ASSGMT	$O(\sqrt{ \mathcal{E}(n) n})$	$O(n)$	$O(\sqrt{ \mathcal{E}(n) n})$
GRID ASSGMT	$O(\mathcal{E}(n))$	$O(n)$	$O(\mathcal{E}(n))$

- Recall, dense $\mathcal{E}(n) \Rightarrow \frac{|\mathcal{E}(n)|}{n} \rightarrow 0$ as $n \rightarrow +\infty$.
- Connectivity regime: $\frac{|\mathcal{E}(n)|}{n} \in O\left(\frac{1}{\log n}\right)$.

Theorem (Stochastic performance)

- n agents, m targets uniformly randomly distributed in $\mathcal{E}(n)$.
- Assume $\mathcal{E}(n)$ is in connectivity regime.
- If $m = n$, then w.h.p. completion time in $O(\sqrt{|\mathcal{E}(n)|})$.
- If $m = n/\log n$ then w.h.p., completion time in $O(1)$.

Stochastic properties of ETSP ASSGMT

- In sparse $\mathcal{E}(n)$:
If $m = n$, then stochastic performance is same as worst case
- In critical or sparse $\mathcal{E}(n)$:
If $m = n / \log n$, then completion time is $O(\log n)$.

Stochastic properties of GRID ASSGMT in connectivity regime

- If $m = cn$ for some $c \in (0, c_{\text{crit}})$, then compltn time is $O(1)$.
i.e., constant factor additional agents \implies for $O(1)$

In **this talk**, introduced:

- a broad class of algorithms for static target assignment;
- asymp. opt. algorithms for **dense and sparse environments**;
- a **sensor based** target assignment problem.

Variations and other problems:

- Nonholonomic vehicles (w/ EF and KS)
- Consistent knowledge assumption
- Related problems
 - ① Targets arriving sequentially/dynamically over time (w/ EF)
 - ② Search and assignment problems
 - ③ Moving targets

- ① S. L. Smith and F. Bullo. Target assignment for robotic networks: Asymptotic performance under limited communication. In *American Control Conference*, pages 1155–1160, New York, July 2007
- ② S. L. Smith and F. Bullo. Target assignment for robotic networks: Worst-case and stochastic performance in dense environments. In *IEEE Conf. on Decision and Control*, pages 3585–3590, New Orleans, LA, December 2007
- ③ S. L. Smith and F. Bullo. Monotonic target assignment for robotic networks. *IEEE Transactions on Automatic Control*, June 2007. Submitted